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Nonlinear optimisation and rational cash flow

Václav Beran and Petr Dlask
Faculty of Civil Engineering, Technical University of Prague, Thakurova, Czech Republic

Abstract

Purpose – The problem of diverse cash flows associated with a construction "project" appear in four progressive cycles. These are the initiating construction cycle and closing deconstruction cycle (devaluation cycle). The effectiveness of any project is given by capitalisation cycle. The optimisation of payback (credit return) cycle is critical for any project.

Design/methodology/approach – For calculate of activity durations, cash flows and even we may use the spreadsheet table as a tool for expression of calculation formulas. This approach may offer a mechanism for answers regarding the sensitivity of manageable parameters (say changes in costs, construction speeds, duration of activity). The problem of optimal capacity expansion of construction work as a time dependent problem is studied in many different applied contexts. Traditional capacity planning usually begins with a forecast of demand on the basis of organisational or technological needs.

Findings – The implementation of a technical project carried out in conditions of high production speeds and low time reserves requires changes in technologies, organisation and preparation of construction. In each specific case, a civil engineer needs to know the economic impacts (the capability of applicable calculations).

Originality/value – It is obvious from the given example, which has the same features as the execution of a series of construction projects in recent years, that the myth of the importance of executing works in large volumes ahead of the deadlines has significant financial consequences.

Keywords Spreadsheet programs, Optimization techniques, Productivity rate, Cash budget, property management

Paper type Research paper

Inspiration

Tradition and experience, have built a whole series of myths into civil engineering. Some are useful and perpetuate the tradition and ethics of the profession, but many are outdated and no longer relevant in the high-speed production conditions of modern construction.

This refers to the following principles (theses):

- To build quickly (under any circumstances) means to build economically. Antitheses (Kant, 1891): we have to vary speed rates according to stage and duration of a project.
- A construction manager fulfilling the earliest possible deadline is a good construction manager. Antithesis: a good manager diversifies production speed at each period of the project he is active in. Earliest possible fulfilment of obligations leads to higher costs for each partner (manufacturer and investors).

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- To build continuously means to build economically. Antithesis: stable speed means higher cost as dynamic change is based on optimisation.
- Cumbersome technologies are disadvantageous. Antithesis: experimentation means new ways and profits.
- Payment for work in progress is a good principle. Antithesis: payment has to diversify according at what progress phase of the project the activity really is placed in.

The problem of diverse cash flows associated with a construction "project" appear in four progressive cycles. These are the initiating construction cycle and closing deconstruction cycle (devaluation cycle) see Figure 1. The effectiveness of any project is given by capitalisation cycle. The payback (credit return) cycle is critical for any project.

The time dependent function of costs $C(t)$ and economical effects $E(t)$ creates the spaces of cumulated gains or losses. The chain of cycles is interrupted by milestones of change from construction in progress to used property in t_0. The pay back cycle (credit return cycle) is limited by break event point (BEP) marked as t_e. The most important point from an economical view is the max difference between $E(t)$ (net benefits) and costs $C(t)$.

Early completion may not be either necessary or economically useful, from the modern construction point-of-view. However, the idea that what is completed can be counted on, has extraordinary strength in some areas of the construction industry, investments and other project with a long life cycle (LLC). The efforts of many contract managers to create a time reserve and to lower the risk of breaching the construction deadline go so far as to perform a series of works earlier than is technically and organisationally necessary. We may observe this tendency in numerous projects.

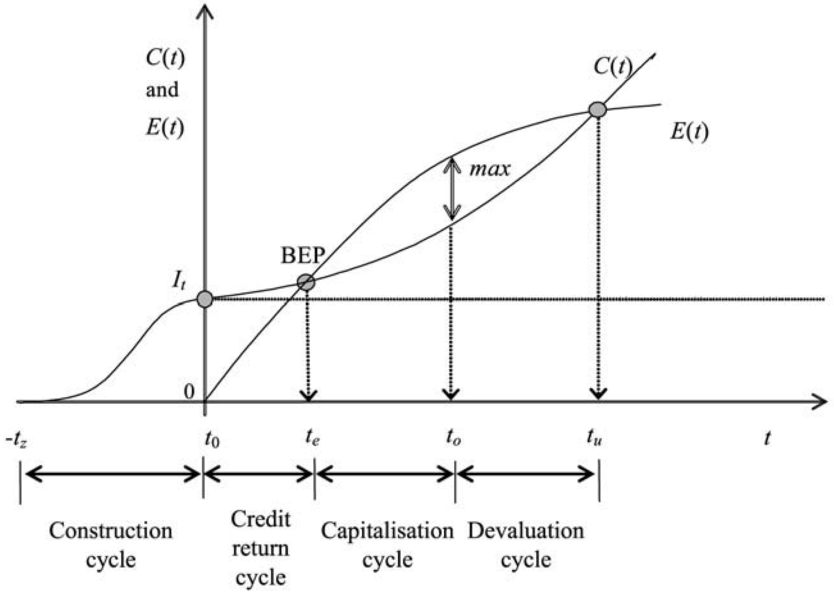


Figure 1.
Rational cash flow and LC layout structure

Textbook examples may contain large and extensive file of projects. The author of this paper will be happy to receive more examples such as the cooling towers of the Temelin nuclear power plant (competing nuclear power plants in the Czech and Austrian government war of words), or the conservation of completed nuclear power stations in Austria. In an effort to engage in expensive construction work (and to create time reserves for the future), the dominant construction feature during decades of construction were the cooling towers

Cooling towers are, however, technologically simple structures that can be added much later in the project. Nevertheless, they were the first technological structure erected on the building site in Temelin (the initial power station design was developed by Energoprojekt Prague in 1985, and the construction of the operating units was launched in 1987). It is easy to get the reality on screen, visit www.cez.cz/presentation/eng/ and after clicking power stations you will see the towers current reality. The conservation of completed nuclear PS in Austria and the use of conventional sections of plant is a similar question. How far they are economic examples of bad construction management is our discussed question. Plant construction started in 1983 (supporting objects) or in 1987 (main PS objects) and had finished with the support of corporation Westinghouse two of proposed four blocks say in the year 2002 (test). The activities time scheduling of large projects are time-dependent and the calculation of economical speed of construction is a question of optimisation in construction policy.

The question mentioned has many forms and application challenges, for example in renovation, pay back strategies, maintenance of existing construction substances, administration of property portfolios and other. Figure 1 shows the scheme of time dependent chain of cycles. However, any simple time scheduling in construction, maintenance and property portfolio schemes needs a calculation that is more sophisticated. The Figure 2 shows some property time-layout for long live cycle (LLC). Even in this scheme, we may observe that the optimal strategy is not only to keep the portfolio. In contrary, it is necessary for maximisation of benefits to vary the intensity in the use of diverse objects, develop maintenance, renovation and even deconstruction. In Figure 2 is visualised LLC fixed price income cash flow and an extract segment of a maintenance and renovation proposal.

The solution is based on a spreadsheet calculation. The spreadsheet is seen as an individual object (structure) that enables a broad range presentation of technical-economical processes.

Spreadsheet and the dynamic schedule

It is possible to get the input scheme of activities duration (partial jobs) by spreadsheet calculation (see Figure 2) from a practical point-of-view. It is practical to calculate activity durations, cash flows and even if some activities are risk conditioned and then transfer the risk to the results. We may see the spreadsheet table as a tool for expression of calculation formulae (Beran, 1985, 1998, 2000). This approach may offer a mechanism for answers regarding the sensitivity of manageable parameters (say changes in costs, construction speeds, duration of activity i (job) will be calculated as Costs(i)/Production Speed(i), or $D_i = c_i/v_i$. We scrutinise the spreadsheet not only as a matrix or a data filled table. A dynamic aspect of calculation is brought into play and $\mathrm{TAB}_{DHProject}$ is a complex description of project that enables calculations, simulations, and parameterisations for the evaluation of potential management changes. A

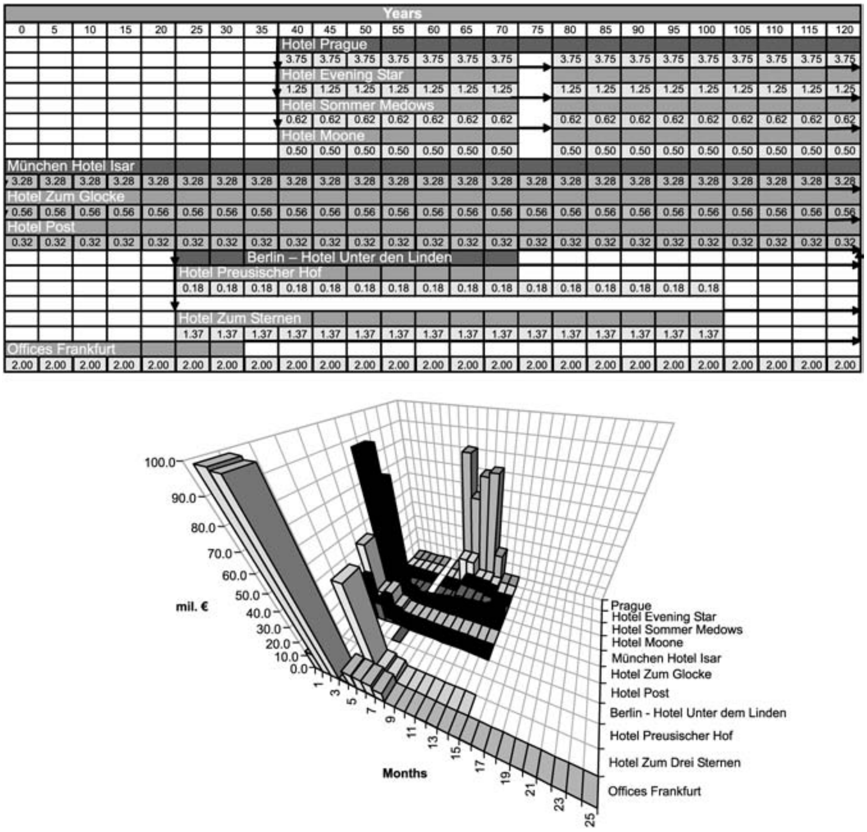


Figure 2.
Inputs for portfolio project and optimised structure of exploitation

symbolic description for deterministic calculated projects is a table developed on implementation of a real functional structure:

$$\mathrm{TAB}_{\mathrm{DHProject}} = N[f(t, Q, \acute{Q}) \Rightarrow D, \therefore Org] \qquad (1)$$

more particulars and applications are in (Beran *et al.*, 2002) available. The results are a composition N.

Generally it is a oriented graph, (network) and in methodology used in (Beran *et al.*, 2002) dynamic schedule (dynamic harmonogram (DH)) involving a duration sets of activities D calculated on the basis of quantities Q given predominantly as costs and production speeds Q'. The structure that reports to organisational structures among activities is the $\therefore Org$. Similar to the calculation of durations D according to (1) play the role in quantifications of durations production dependences $\therefore D$. Let us say organisational and technological connections of activities such as start-start, end-start, end-end; combined connections like the critical activity approach, minima condition, maxima condition, and other logical connections (see 6) of type if-then. We write a more extensive notation (2) for this purpose as:

$$\mathrm{TAB}_{\mathrm{DHProject}} = N[D = f(t, Q, \acute{Q}, \therefore D), \therefore Org] \tag{2}$$

The commercial SW platforms offer a range of products for time sequencing or even project management calculations. The comprehensive open approach in this paper was developed on a spreadsheet platform. This enables the calculation of time and costs and furthermore enumeration of risks as given schematically in (3). An extensive example set and spreadsheets collection is the main topic in DH (Beran *et al.*, 2002) and CD enclosed.

The practical management decision needs further information about risk and sensitivity to proposed managerial changes. The concise notation of the problem is given in (3) and (3a):

$$\mathrm{TAB}_{\mathrm{DHProject}} = N[D = f(Q|risk, \acute{Q}|risk, \therefore D_{network}), \therefore Org|risk] \tag{3}$$

The parameterisation paves the way to limit management intentions and achieves deeper economic reasoning. The parameterisation calculation is time consuming and graphical presentations are desirable. The shortcut presented in (3a) round up the problem to the main corner stones:

$$\mathrm{TAB}_{\mathrm{DHProject}} = N[D = f(Q|param, \acute{Q}|param, \therefore Dnetwork), \therefore Org|param] \tag{3a}$$

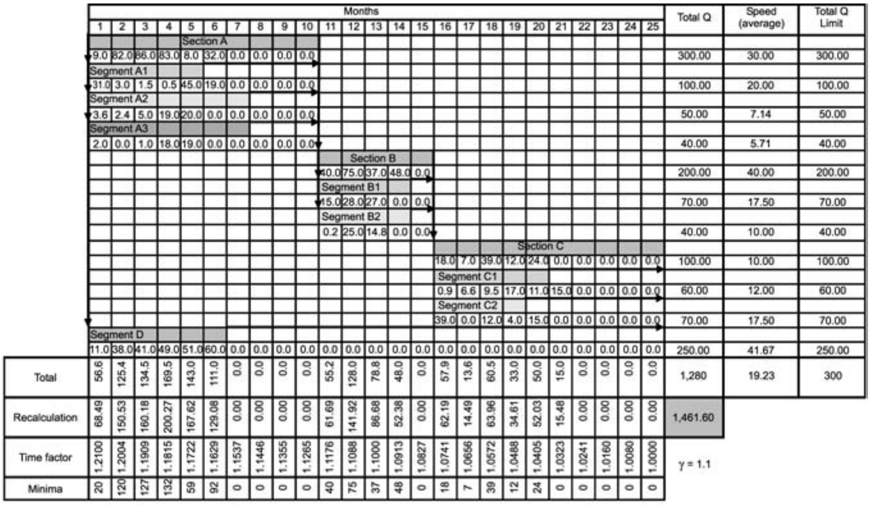
The presented paper is an outgoing extension from expressions (1) to (3a). In this paper, however, we will follow the relative more sophisticated paths of optimisation. Based on existing limits of management possibilities (1) to (3a) we will develop optimal solutions. Such solutions are the boundary lines for interest of construction enterprise management on one and financial body management on other side. This paper will show that the optimal solutions for mentioned players on actual construction market have only one fair solution. This is the equity of slow production (construction) speed with respect to limits given for optimisation in spite of minimisation or maximisation. In shortcut it is:

$$\begin{aligned} &\min C\{\mathrm{TAB}_{\mathrm{DH\,investment}}|production\ speeds\} \\ &\quad = \max C\{\mathrm{TAB}_{\mathrm{DH\,contractor}}|production\ speeds\} \end{aligned} \tag{3b}$$

where optimisation on the basis of min/max $C\{\bullet\}$ presents a cost functions including time factor. Inputs TAB are presented by LC's given in Figure 1 or upper part of Figures 2 and 3.

Formulation of the optimisation problem – pragmatic part

A construction project, investment project, portfolio structure or maintenance project (see Figure 1) is understood as $\mathrm{TAB}_{\mathrm{project}}$ in this paper. The presentation of an optimised portfolio time layout is the challenge scope of Figure 2. The presentation of optimised construction production progress we see in Figure 3. In this example, movable segments conduct critical paths of time-sequenced activities A, B, C. The input origin structure of Figure 3 is result of an earlier mentioned spreadsheet calculation. The correct time and costs layout are the result. The main defect of most commercial products for time scheduling is the precondition of constant production speed for activities. Vice versa, for sophisticated time factor are dependent all



	Months																									Total Q	Speed (average)	Total Q Limit
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25			
Section A	9.0	82.0	86.0	83.0	8.0	32.0	0.0	0.0	0.0	0.0																300.00	30.00	300.00
Segment A1	31.0	3.0	1.5	0.5	45.0	19.0	0.0	0.0	0.0	0.0																100.00	20.00	100.00
Segment A2	3.6	2.4	5.0	19.0	20.0	0.0	0.0	0.0	0.0	0.0																50.00	7.14	50.00
Segment A3	2.0	0.0	1.0	18.0	19.0	0.0	0.0	0.0	0.0	0.0																40.00	5.71	40.00
Section B											40.0	75.0	37.0	48.0	0.0											200.00	40.00	200.00
Segment B1											15.0	28.0	27.0	0.0	0.0											70.00	17.50	70.00
Segment B2											0.2	25.0	14.8	0.0	0.0											40.00	10.00	40.00
Section C																18.0	7.0	39.0	12.0	24.0	0.0	0.0	0.0	0.0	0.0	100.00	10.00	100.00
Segment C1																0.9	6.6	9.5	17.0	11.0	15.0	0.0	0.0	0.0	0.0	60.00	12.00	60.00
Segment C2																39.0	0.0	12.0	4.0	15.0	0.0	0.0	0.0	0.0	0.0	70.00	17.50	70.00
Segment D	11.0	38.0	41.0	49.0	51.0	60.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	250.00	41.67	250.00
Total	56.6	125.4	134.5	169.5	143.0	111.0	0.0	0.0	0.0	0.0	55.2	128.0	78.8	48.0	0.0	57.9	13.6	60.5	33.0	50.0	15.0	0.0	0.0	0.0	0.0	1,280	19.23	300
Recalculation	68.49	150.53	160.18	200.27	167.62	129.08	0.00	0.00	0.00	0.00	61.69	141.92	86.68	52.38	0.00	62.19	14.49	63.96	34.61	52.03	15.48	0.00	0.00	0.00	0.00	1,461.60		
Time factor	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000	γ = 1.1		
Minima	20	120	127	132	59	92	0	0	0	0	40	75	37	48	0	18	7	39	12	24	0	0	0	0	0			

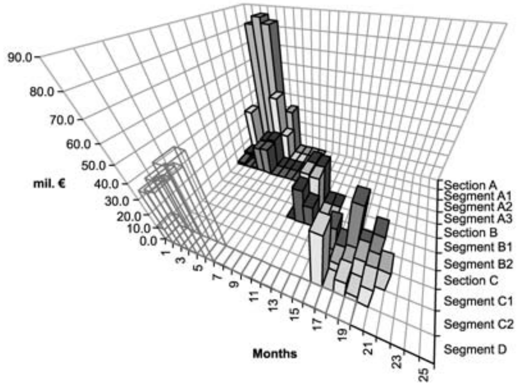


Figure 3.
Completed data for optimal earliest starts and speed limit $Q'_{project} = 300$t. €/months, reconstruction project proposed from construction consortium

rescheduling bringing an extra profit in return. The solution is optimisation, based on the solver integrated in most spreadsheet packages. The result is a re-location of cash flows and optimal timing of movable activities.

There exist other reasons to look after for optimal solutions. Such a solution presents a limit in the potential that we might achieve. The distance between the pragmatic management proposal optimal solution presents a measure of rational utility of a management decision. Let's say that any managerial order is constructed on the knowledge of a process describing reality P technically described by means of $TAB_{project}$ and a management model process L developed as maxmin $TAB_{project}$. Then the managerial order is given as $\phi_i(t, \mathrm{P}, \mathrm{L})$. The difference of expected utility of these two results $U_i[\phi_i(t, \mathrm{P}, \mathrm{L})]$ is the rationality measure of a managerial order under given restrictions.

The risk analysis calculation is a relatively a sophisticated supporting calculation, however not further followed in this paper. Another approach, parameterisation, is able to mark-up the potential space for decision-making. However, this approach in empirical solutions-space, we will not follow.

This paper deals with optimisation. The calculation of optimal production speed leads to a declaration of optimal duration of a project as whole optim$T_{\mathrm{End},}$ and optimal duration of particular jobs or activities. Total volumes of activities (cost) are taken as an optimisation restriction. Optimisation criteria are min/max of cost recalculated by time factor. Optimisation creates for management decisions new safer horizons and new limits.

In Figure 3 we may follow results for the earliest possible timing. The consequence is the (maximal) finish float of activities and low-lying risk to defect the start of next critical activity (read section B in Figure 3). The optimal earliest possible finish of activities was limited by max speed 300 t. €/months. The sum of project costs (without time factor) is given by 1280 t. € (see value 300 in line marked as total in Figure 3).

For the bank loan rate of $i = 0.10$ the value of construction money increases during 25 months of construction time to 1461.60 t. €. There are many ways how to recalculate cost to the future costs. In the example given in the Figure 3 we use a recalculation of production speed $Q'(t)$ by means of power function γ^t. The present value of total production speed $Q'_{\mathrm{PV}}(t)$ is given as $Q'_{\mathrm{PV}}(t) = Q'(t)\gamma^{\mathrm{t}}$, where time increases from the construction start in $t = -25$ (left side of the Figure 3) to $t = 0$ in the project finish at the right side of the same figure. A similar result may be given by an exact calculation used in financial calculations as $Q_{\mathrm{PV}}'(t) = Q'(t)1/(1-i)^t$. The alternative calculation by means of relation $Q_{\mathrm{PV}}'(t) = Q'(t)e^{it}$ may offer also an acceptable solution for optimisation. Independent from the form and level of the loan rate, the optimal solution will be always the same if any time factor was used.

Time factor $\gamma = 1.10$ causes cca 14.19 per cent increase of total investment project costs. This percentage indicates, roughly, credit costs and unproductive frozen investment assets. The improved technology and organisation of construction schedule is, from this point-of-view, economically desirable and feasible (see data in Figure 4).

We will profit, from a mechanical point-of-view, of possible shifts (say earliest and latest possible start and finish. If segments (jobs activities) A1, A2 and A3 are slid to the end of A section and similar segments B1, B2, C1, C2, C2 and D to the end of their sections, the total latest cost flow will be gained. If we recalculate cash flow by the time factor given in cash flow for latest starts the total value of money decrease. Moreover, if

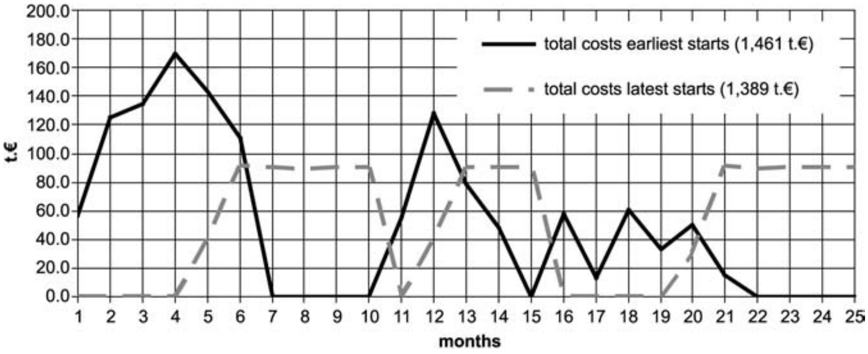


Figure 4.
Comparison of total and in time fixed cash flow costs for project schedule based on data of Figure 3

we are able to create a dynamic shape of activities (see Figure 4), there is more space for effective rationalisation of cash flow. Cost is 1389.45 t. €. Optimisation may pave the way to this sophisticated efficiency. Sophistication is limited. We understand it only in terms that an optimal solution is not intuitively available.

In comprehensive thinking, there are usually clear reasons responsible for the high cost of projects:

- long construction time;
- bad strategy of construction work sequence; and
- high cost of money (interest rate).

The long construction times of the projects and losses during the unproductive boundary of resources are given by the sum of resources spent, multiplied by γ^t, where γ is total commercial interest rate (rate usually configured from the credit interest rate). The construction of γ may differ for diverse projects. It usually includes risk rate and entrepreneurial profit. For a commercial case it might be compounded by components mentioned by $\gamma = 0.10 + 0.04 + 0.03$. The losses from frozen capital due to slow production speed for say $t = 15$ years construction time raise $0.17^{15} \Rightarrow 2.55$ times capital spent for ordinary cash flow spent in construction. The mentioned case of the power plant shows an example where the technology had a wrong sequence. This situation is predetermined by low production capacity (low ability to create high construction speeds). There are also other reasons why construction management has shifted to other then economic rules. Reliability of finishing date will be the other reason. Wrong contracting conditions, instead of lump cost, paying according to work done, without differentiation of work due to time break down of structures. Generally, a fair break down of advantages and risks of the contracting and contracted subject might lead to more respectable economic situations.

What is the main question? How it will be possible to create a rational (optimal) strategy for cash flow during the construction period? The interest rate might lead us to question the robustness of this constant. However, the level of interest is not what moves this model forward but the optimisation of cash flow proposed for different activities (sections and segments in Figure 3). The time we have for construction is not continuous. Construction time is discrete time linked to limited available resources. We have to develop strategies for the starting and finishing periods of investment activities that have to be established. Discussion about the level of interest rate, which will be used for calculation, is always inviting and appealing.

Private business looks to protect its right to get approximately the same level of revenues as government take (see the left and right hand side of formula (6)).

$$\text{Capital interest rate} + \text{Risk rate} + \text{Entrepreneurial profit rate} \approx \text{GDP} \qquad (4)$$

From an economic point-of-view, it is necessary to see the equation as a balance of the free entrepreneurial profit calculation on the left hand side of equation (4) and a governmental expectation or deal on money spent on investment on the right hand side. Nevertheless, the main reason to fix any rate of interest is to get data for the calculation of an optimisation strategy. Optimal strategy shows how to behave in the period from the start to end of the construction time or investment assets. The correct value of γ seems to be a more comprehensive question. Nevertheless, there is no relation between strategy (structure of construction works) and the robustness of interest rate (time factor) build into optimisation.

For an optimal strategy it is irrelevant how considerable the value γ really is. This value only makes bigger the penalty if management does not keep to an optimal strategy in their long-term time schedule. The structure of optimal cash flow does not change.

In the technologies and organisational processes for industrial buildings, railways and road reconstruction, public utilities and housing developments there are assembly procedures that are very appropriate for the given purpose. Complicated research-, development-, innovation-projects are all very good aspects of these kinds of applications. However, the cooling towers syndrome mentioned in the inspirational introductory may also be found there. The organisational process in the construction industry seems to favour extensive early completion of parts of projects. The application of JIT-type procedures would certainly be economically more suitable. The investors of large investments and all public invested money may be spent more efficiently if an optimisation strategy can be calculated.

A generalisation of time dependent capacity (TDC)

The problem of optimal capacity expansion of construction work as a time dependent problem is studied in many different applied contexts (Mulvey *et al.*, 1995). Traditional capacity planning usually begins with a forecast of demand on the basis of organisational or technological needs. Planning and scheduling has for many years been the dominant approach by the Central European management methodology. New approaches in adopting a more productive methodology seem to be needed. Modern management of time dependent capacity expansion enables applications in production planning, strategic planning, inventory control, and network design. Applications to telecommunications are published by Laguna (1998).

The was TDC problem consists of finding the combination of activities j ($j = 1, 2, \ldots, N$) with price p_j and demand efficiency $c_{j,}$ that should be employed in each time period t ($t = 1, \ldots, T$). The limitations are given by the total demand (capacity) D_t at a minimum cost.

Then, the problem becomes:

$$\min \sum_{t=1}^{T} \sum_{j=1}^{N} p_j \gamma^{t-1} x_{jt} \tag{5}$$

subject to:

$$\sum\nolimits_{t=1}^{T} \sum\nolimits_{j=1}^{N} c_j x_{jt} \geq D_t \tag{6}$$

for all t where:

$$x_{jt} \geq 0 \tag{7}$$

is production speed for all j and t. Further $\gamma = 1 + i$ enables the recalculation of production speed values on future value, by means of a discount factor $(0(i(1)$ for all x_{jt} for activities j in time t.

The structure of the production speed is quite variable. Table I shows a general example of this interpretation. Demand D_t may be structured not only as to t as a particular time period, but also to demand blocks related to different activities j and even to blocks of technologically (∴D) or organisationally (∴Org) related activities mentioned in (1), (2), (3).

If the matrix of variables in time t, where $t = (1, 2, 3, \ldots, T)$ is assigned for particular scenarios, where ($s = 1, 2, \ldots, S$), say as z_{ts}, the problem becomes in (Laguna, 1998) form:

$$\min F(\boldsymbol{x}) = \sum_t \sum_j p_j \gamma^{t-1} x_{jt} + w\rho(\pi_s, z_{ts}) \tag{8}$$

subject to:

$$\sum_t \sum_j c_j x_{jt} + z_{ts} \geq D_{ts} \; \forall t, s, \tag{9}$$

$$x_{jt} \in (0,\ 1,\ 2, \ldots) \; \forall j, t, \tag{10}$$

$$\mathrm{z}_{\mathrm{ts}} \geq 0 \; \forall t, s, \tag{11}$$

where w is the weighting factor and ρ is the function of negative demand consequences related to the unmet demand z_{jt} and probability π_s in the range of scenarios.

Demand D_{ts} will be presented with an uncertainty component z_{jt}, see equation (9). This represents an imaginary demand associated with the risk of shortage of capacity with a probability π_s related to scenario s at each period t. Function ρ may take many forms. It usually reflects the risk attitude of the decision maker. The risk may be associated with the probability of shortage of capacity, the risk of extra costs or the risk of lack of quality if the production speed exceeds certain limits. Further applications and interpretations are possible.

The effect of JIT – optimisation of dynamic harmonogram DH

Let us leave aside considerations, theories and detailed analysis for a moment. Let us direct our attention to a pure economic problem. How can the possible effects of optimisation be applied to the construction industry and investment strategies?

We can look for comparison at the definition of management (Von Neumann and Morgenstern, 1947), there are two processes created in a time-space. First one describes reality (or takes data from this space) and second one describes controlled models as a compression of reality.

Table I.
General scheme of a production structure

	$t = 1$	$t = 2$	$t = 3$	...	$t = \mathrm{T}$
Activity 1	x_{11}	x_{12}	x_{13}	...	$\mathrm{x}_{1\mathrm{T}}$
Activity 2	x_{21}	x_{22}	x_{23}	...	$\mathrm{x}_{2\mathrm{T}}$
	...	...	...	...	...
Activity N	x_{N1}	x_{N1}	x_{N1}	...	x_{N1}
	D_1	D_2	D_3		D_T

Let us assume that we are implementing and completing a simplified, extensive reconstruction of a transportation project (infrastructure). The critical production activities are sections A, B, C (see table in Figures 3 and 5). In addition to these activities, there are some production activities that may be freely movable. The possible rescheduling for different production speeds of these activities are presented further in Figures 5 and 6. Please pay attention to possible scheduling of the segments inside the technological sections and total capacity and average production speed of activities jobs.

Let us now see and evaluate the overall effect of construction execution and calculate the cost of the necessary operation credits for carried out works as they were proposed. For the sake of simplicity we assume that there will not be any delays in payments (in invoice payments, salary payments, payments to contractors and subcontractors). The basic scheme is only limited in its flexibility in terms of time. We will investigate the advantages to be gained through optimisation under various conditions. Relocating the non-fixed activity segments provide the first guidance.

	Months																									Total Q	Speed (average)	Total Q Limit
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25			
					Section A																							
	0.0	0.0	0.0	0.0	9.0	82.0	86.0	83.0	8.0	32.0																300.00	30.00	300.00
	Segment A1																											
	0.0	0.0	0.0	0.0	31.0	3.0	1.5	0.5	45.0	19.0																100.00	20.00	100.00
	Segment A2																											
	0.0	0.0	0.0	0.0	0.0	3.6	2.4	5.0	19.0	20.0																50.00	7.14	50.00
	Segment A3																											
	0.0	0.0	0.0	0.0	0.0	2.0	0.0	1.0	18.0	19.0																40.00	5.71	40.00
												Section B																
											0.0	40.0	75.0	37.0	48.0											200.00	40.00	200.00
											Segment B1																	
											0.0	0.0	15.0	28.0	27.0											70.00	17.50	70.00
											Segment B2																	
											0.0	0.0	0.2	25.0	14.8											40.00	10.00	40.00
																				Section C								
																0.0	0.0	0.0	0.0	18.0	7.0	39.0	12.0	24.0	0.0	100.00	10.00	100.00
																Segment C1												
																0.0	0.0	0.0	0.0	0.9	6.6	9.5	17.0	11.0	15.0	60.00	12.00	60.00
																Segment C2												
																0.0	0.0	0.0	0.0	0.0	39.0	0.0	12.0	4.0	15.0	70.00	17.50	70.00
	Segment D																											
	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	11.0	38.0	41.0	49.0	51.0	60.0	250.00	41.67	250.00
Total	0	0	0	0	40	90.6	89.9	89.5	90	90	0	40	90.2	90	89.8	0	0	0	0	29.9	90.6	89.5	90	90	90	1,280	19.23	300
Recalculation	0.00	0.00	0.00	0.00	46.89	105.36	103.72	102.44	102.20	101.39	0.00	44.35	99.22	98.22	97.22	0.00	0.00	0.00	0.00	31.11	93.52	91.66	91.44	90.72	90.00	1,389.45		
Time factor	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000	γ = 1.1		
Minima	0	0	0	0	9	82	86	83	8	32	0	40	75	37	48	0	0	0	0	29	45	80	61	75	60			

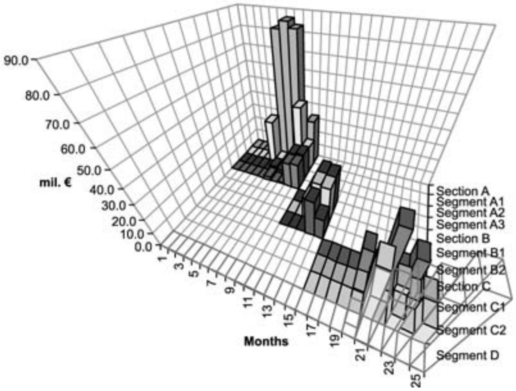
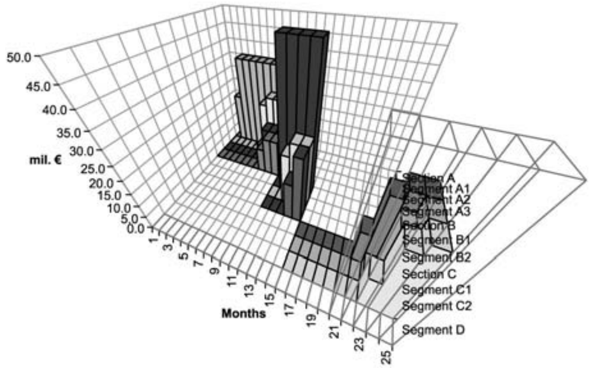


Figure 5.
Deadlines and costs for latest possible end speed 300 t. €/month



Total Q	Speed (limit)	Total Q Limit	Speed (average)
300.00	35.00	300.00	30.00
100.00	25.00	100.00	20.00
50.00	15.00	50.00	7.14
40.00	15.00	40.00	5.71
200.00	50.00	200.00	40.00
70.00	25.00	70.00	17.50
40.00	30.00	40.00	10.00
100.00	30.00	100.00	10.00
60.00	30.00	60.00	12.00
70.00	25.00	70.00	17.50
250.00	50.00	250.00	41.67
1,280	30.00	300	19.23

Total	0	20	35	35	35	38.034	61.966	85	90	90	3E-14	50	70	89.652	100.35	0	0	0	0	2E-08	50	62.682	95.633	131.76	139.92	1,280
Recalculation	0.00	24.01	41.68	41.35	41.03	44.23	71.49	97.29	102.20	101.39	0.00	55.44	77.00	97.84	108.64	0.00	0.00	0.00	0.00	0.00	51.61	84.19	97.16	132.82	139.92	1,389.29
Time factor	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000	γ = 1.1

Figure 6.
Deadline production speeds and total time dependent costs of construction project

The optimal schedule of production activities to the latest ends is illustrated in Figure 5. The production activities are carried (finished) out at the latest possible starts and ends. In the given case, the deadlines and financial payments take into account some restricting conditions. In this case it is total production speed 300 t. €/months (right corner of line total). Other more sophisticated limits are diversified production speeds for individual production activities (jobs), taking technological and organisational considerations into account. The aim is to minimise the costs, including interest payments, for completing the construction. The optimum solution is in many ways surprising. Sophisticated changes, using optimisation, might lead in many cases to a radical drop in the total costs. Even if no profit could be achieved, there is a range of possible managerial manipulations that could, if skilfully exploited in terms of JIT, produce cost savings.

Each optimisation can lead to further possible improvements over the primary optimisation and shows which production sources (limits) can be incorporated into the restricting condition limits. Further improvements to the solution are possible by means of shadow prices. A more detailed analysis can show under what conditions production sources (production speed or cash flow of activities) if increased creates further improvements of the solution.

If we compare the structure of activities in Figures 3 or 4 with the non-optimised solution and look at Table II, where columns 2 to 4 give the main parameters of the task in Figures 3 and 4 we will see the following facts:

Table II.
Comparison of solutions for total speed limitation Q′project = 300t. €/month

Actions total (in t. €)	Earliest possible deadlines	Latest possible deadlines
Total costs (no interest rate IR calculated)	1280.00	1280.00
Including bank loans (IR = 10%)	1461.60	1389.45
Average production speed	19.23	19.23
Total production speed limit	300.00	300.00
Proportion, incl. IR (%)	100.00	95.06

- duration times are changed for all production activities;
- production speeds are changed in the course of optimisation criteria to earliest or latest execution;
- the total duration of the construction work and segment has changed (the proposal created new time reserves to cover risks in connections with production speed);
- the main production processes will be speeded up, the duration of the active project processes are shorter;
- the average production speed of the construction work is the same as the original production speed; and
- a decrease in total costs to 95.06 per cent in comparison to the initial solution.

Other scenarios could also be presented. The main outcome of the whole task is an increase in production speeds and a reduction of time margins (floats). The overall effect is in essence a change in the organisation of project completion.

The outlined example shows a method for completing a floor (arrangements of surfaces and connecting parts of structures), i.e. technology that can substantially influence the final effect of the construction. A 6 per cent reduction in the cost of a component and its effect on the profitability of the project is a considerable argument and motivation for technological and organisational changes in the direction of JIT.

Speed (limit) average	Optimal solution for latest costs t. €	Optimal solution for earliest costs t. €
50	1472.15	1381.20
45	1458.36	1382.88
40	1461.97	1383.73
35	1461.57	1384.61
30	1459.36	1386.65
25	1460.27	1395.06
20	1456.26	1403.89
19, 45	1455.57	1405.00

Table III. Production speed and minimal costs for $i = 0.10$

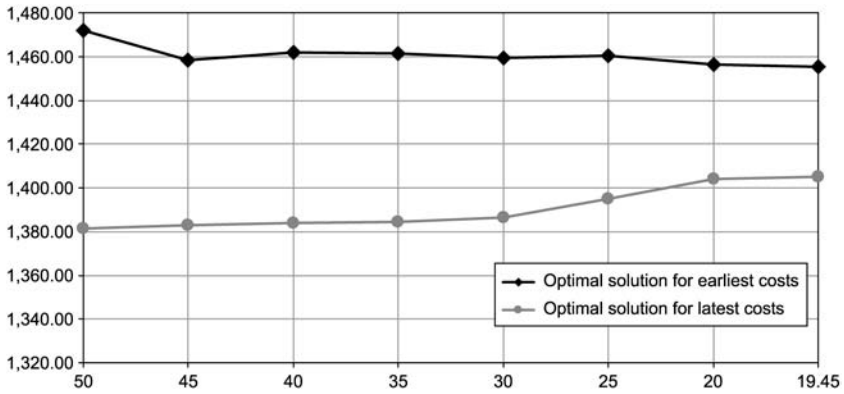


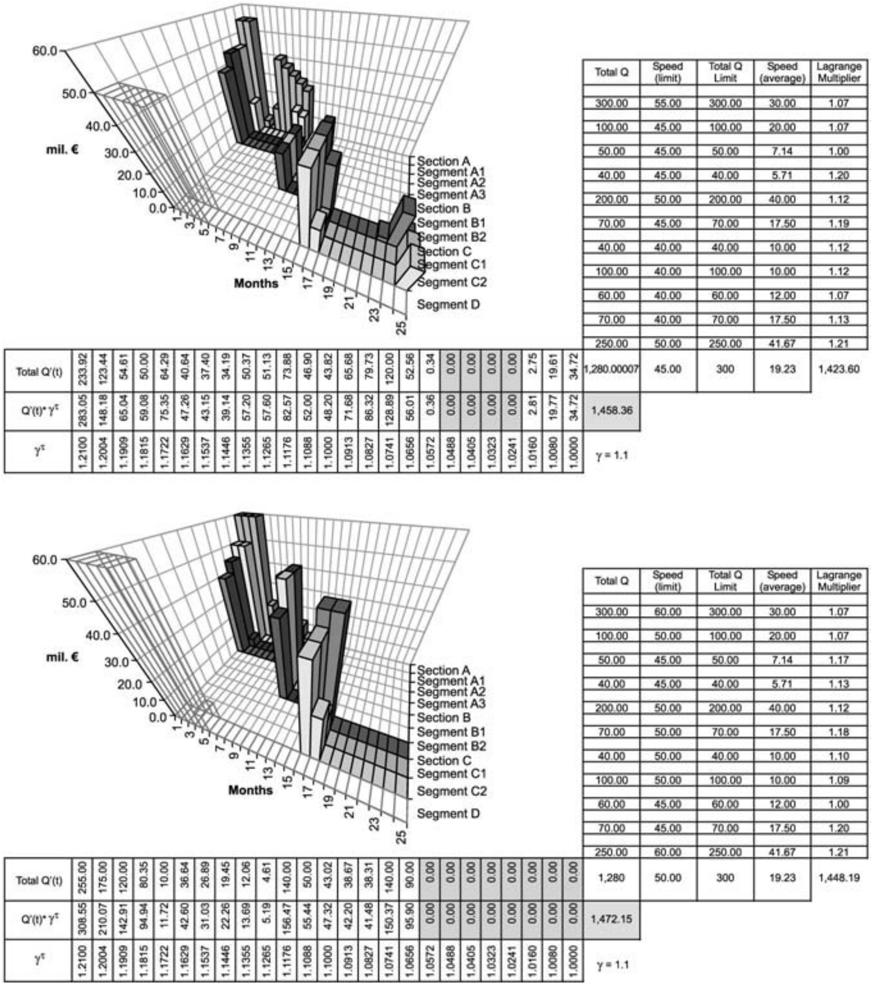
Figure 7. Dependence of optimal solution and speed limit

Model construction – further generalisations for production speeds

An interesting question is how far an optimal solution is dependent on production speeds and what may be the optimal construction duration of an investment project. The production speed is seen as a given limit for every construction activity. An example of the model can used theory in (Holling, 1978).

A test is given in Figure 6. Particular speed limits of activities are taken from Figure 5 with minor exception. Solution is practically the same as in Figure 5. The optimisation function remains in the shape as before. Table III shows the optimisation results speeds in range (19, 45, 50).

As Table III indicates there is a strong dependence between the first column describing achieved average production speed and the last column – the minimal costs of projects. More illustrative are data in Figure 7. Total cost of projects grows in correlation with the rate of exploited production speed.



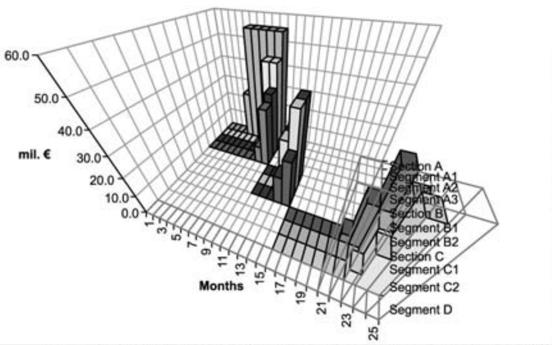
Total Q	Speed (limit)	Total Q Limit	Speed (average)	Lagrange Multiplier
300.00	55.00	300.00	30.00	1.07
100.00	45.00	100.00	20.00	1.07
50.00	45.00	50.00	7.14	1.00
40.00	45.00	40.00	5.71	1.20
200.00	50.00	200.00	40.00	1.12
70.00	45.00	70.00	17.50	1.19
40.00	40.00	40.00	10.00	1.12
100.00	40.00	100.00	10.00	1.12
60.00	40.00	60.00	12.00	1.07
70.00	40.00	70.00	17.50	1.13
250.00	50.00	250.00	41.67	1.21
1,280.00007	45.00	300	19.23	1,423.60
1,458.36				
γ = 1.1				

Total Q′(t)	233.92	123.44	54.61	50.00	64.29	40.64	37.40	34.19	50.37	51.13	73.88	46.90	43.82	65.68	79.73	120.00	52.56	0.34	0.00	0.00	0.00	0.00	2.75	19.61	34.72
Q′(t)* γ^τ	283.05	148.18	65.04	59.08	75.35	47.26	43.15	39.14	57.20	57.60	82.57	52.00	48.20	71.68	86.32	128.89	56.01	0.36	0.00	0.00	0.00	0.00	2.81	19.77	34.72
γ^{τ}	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000

Total Q	Speed (limit)	Total Q Limit	Speed (average)	Lagrange Multiplier
300.00	60.00	300.00	30.00	1.07
100.00	50.00	100.00	20.00	1.07
50.00	45.00	50.00	7.14	1.17
40.00	45.00	40.00	5.71	1.13
200.00	50.00	200.00	40.00	1.12
70.00	50.00	70.00	17.50	1.18
40.00	50.00	40.00	10.00	1.10
100.00	50.00	100.00	10.00	1.09
60.00	45.00	60.00	12.00	1.00
70.00	45.00	70.00	17.50	1.20
250.00	60.00	250.00	41.67	1.21
1,280	50.00	300	19.23	1,448.19
1,472.15				
γ = 1.1				

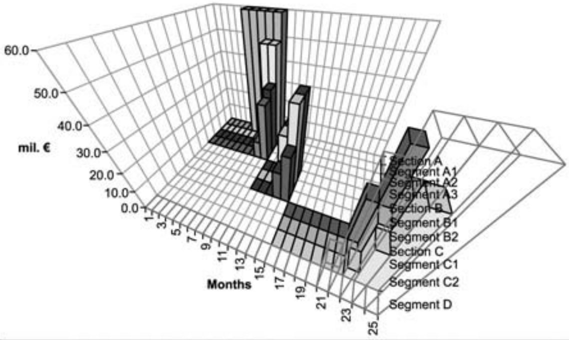
Total Q′(t)	255.00	175.00	120.00	80.35	10.00	36.64	26.89	19.45	12.06	4.61	140.00	50.00	43.02	38.67	38.31	140.00	90.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Q′(t)* γ^{τ}	308.55	210.07	142.91	94.94	11.72	42.60	31.03	22.26	13.69	5.19	156.47	55.44	47.32	42.20	41.48	150.37	95.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
γ^{τ}	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000

Figure 8.
Optimal solution max duration for production speeds as cca $\frac{1}{4}$ of activity volume average of speeds is $Q' = 45$ and $Q' = 50$t. €/month (cost = 1458.36 t. € and 1472.15 t. €, for interest rate $i = 0.1$)



Total Q	Speed (limit)	Total Q Limit	Speed (average)	Lagrange Multiplier
300.00	55.00	300.00	30.00	1.02
100.00	45.00	100.00	20.00	1.02
50.00	45.00	50.00	7.14	1.00
40.00	45.00	40.00	5.71	1.14
200.00	50.00	200.00	40.00	1.09
70.00	45.00	70.00	17.50	1.17
40.00	40.00	40.00	10.00	1.08
100.00	40.00	100.00	10.00	1.09
60.00	40.00	60.00	12.00	1.02
70.00	40.00	70.00	17.50	1.14
250.00	50.00	250.00	41.67	1.14
1,280.00007	45.00	300	19.23	1,389.37

Total Q′(t)	0.00	0.00	0.00	0.00	25.00	55.00	55.00	65.00	125.31	164.69	0.00	50.00	50.00	93.44	116.56	0.00	0.00	0.00	0.00	0.00	50.00	55.03	82.60	135.05	157.32	
Q′(t)* γ^{τ}	0.00	0.00	0.00	0.00	29.30	63.96	63.45	74.40	142.29	185.53	0.00	55.44	55.00	101.97	126.20	0.00	0.00	0.00	0.00	0.00	51.61	56.36	83.92	136.13	157.32	1,382.88
γ^{τ}	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000	$\gamma = 1.1$



Total Q	Speed (limit)	Total Q Limit	Speed (average)	Lagrange Multiplier
300.00	60.00	300.00	30.00	1.02
100.00	50.00	100.00	20.00	1.02
50.00	45.00	50.00	7.14	1.03
40.00	45.00	40.00	5.71	1.13
200.00	50.00	200.00	40.00	1.09
70.00	50.00	70.00	17.50	1.13
40.00	50.00	40.00	10.00	1.08
100.00	50.00	100.00	10.00	1.09
60.00	45.00	60.00	12.00	1.02
70.00	45.00	70.00	17.50	1.14
250.00	60.00	250.00	41.67	1.14
1,280	50.00	300	19.23	1,381.13

Total Q′(t)	0.00	0.00	0.00	0.00	0.00	60.00	60.00	60.00	135.28	174.72	0.00	50.00	50.00	93.42	116.58	0.00	0.00	0.00	0.00	0.00	18.57	51.43	90.55	150.58	168.87	
Q′(t)* γ^{τ}	0.00	0.00	0.00	0.00	0.00	69.77	69.22	68.67	153.62	196.82	0.00	55.44	55.00	101.95	126.22	0.00	0.00	0.00	0.00	0.00	19.17	52.67	92.00	151.78	168.87	1,381.20
γ^{τ}	1.2100	1.2004	1.1909	1.1815	1.1722	1.1629	1.1537	1.1446	1.1355	1.1265	1.1176	1.1088	1.1000	1.0913	1.0827	1.0741	1.0656	1.0572	1.0488	1.0405	1.0323	1.0241	1.0160	1.0080	1.0000	$\gamma = 1.1$

Figure 9.
Optimal solution for production speed as cca 1/8 of activity volume average of speeds is $Q' =$ 45 and 50 t. €/month (cost = 1382.88 t. € and 1381.20 t. € for interest rate $i = 0.1$)

What is interesting is the structure of activities. According to production speed, the time and cash flow structure of construction work changes. We may follow the results for the speed proposed on diverse the levels of total quantity (volume) of project activity.

It is interesting, that the structure of activities will not change if we increase economic pressure by means of interest rate γ^t. If we recalculate Figures 8 or 9 for an interest rate ten times lower than in the original, we will get the same structural time propositions and cash flow propositions as we had in origin. It is surprising that grading compression to a finishing deadline does not exist. There is a plenty of academic discussion about the interest levels that would be necessary or needed for the development of an economic calculation (in terms of efficiency and decision-making). In our case we are looking for the optimal duration and optimal cash flow schedule of construction work. The interest rate is irrelevant if only any formulation of $\gamma > 0$ exists.

For the optimal solution given in Figure 9, there are costs designed at the interest rate $i = 0.10$. For an interest rate ten times lower there is the same structural solution as given in Figure 9, but transaction cost of the present value of the criteria function is significant lower.

Conclusion

The implementation of a technical project carried out in conditions of high production speeds and low time reserves requires changes in technologies, organisation and preparation of construction. In each specific case, a civil engineer needs to know the economic impacts (the capability of applicable calculations). The next important factor in the preparation and choice of management and organisation is the ability to calculate the risks inherent in the chosen technology (Beran, 1985; Beran and Macek, 1998, 2000). It is obvious from the given illustrative example, which has the same features as the execution of a series of construction projects in recent years, that the myth of the importance of executing works in large volumes ahead of the deadlines has significant financial consequences. The interest rate applied here (10 per cent) is not relevant for existing commercial conditions, but there is no ultimate dependence of optimal structure of activities and the corresponding interest rate.

It is very probable that wherever construction work has to be, or was, carried out at a loss or at a low profit, bad time management and cash flow scheduling played a significant role in the economic results.

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Corresponding author

Václav Beran can be contacted at: beran@fsv.cvut.cz